Technical Comments

VTOL Aircraft Dynamics

ROBERT L. SWAIM*
Purdue University, Lafayette, Ind.

IT is well known to stability and control analysts that the small perturbation equations of motion describing the six-degree-of-freedom rigid body dynamics of conventional aircraft are as given by Eqs. (1–6). These apply when stability axes are used and the trim condition is wings level, horizontal flight. The left-side terms involve inertial forces and moments and gravity forces, whereas the right sides contain perturbational aerodynamic forces and moments.

$$m[\dot{u} + g\theta] = F_x \qquad (1)$$

$$m[\dot{w} - U_0 q] = F_z \qquad (2)$$

$$I_y \dot{q} = M_y \qquad (3)$$

$$m[\dot{v} + U_0 r - g\phi] = F_y \qquad (4)$$

$$I_x \dot{p} - I_{xz}\dot{r} = M_x \qquad (5)$$

$$I_z \dot{r} - I_{xz}\dot{p} = M_z \qquad (6)$$

The $-mU_0q$ and mU_0r terms in Eqs. (2) and (4), respectively, are inertial forces due to the q and r angular pitching and yawing velocities. U_0 is the steady state or trim velocity of the aircraft along the horizontal flight path. However, for conventionally flying aircraft, in the absence of an appreciable mean head wind, U_0 is also the relative velocity between the aircraft and air mass and appears in the aerodynamic terms through the dynamic pressure $\frac{1}{2}\rho U_0^2$.

Now, for conventionally flying aircraft with a strong head wind, and especially for hovering VTOL aircraft, the U_0 in the two inertial terms referred to above is considerably less than the U_0 in the dynamic pressure. Or in more familiar terms, the flight path velocity (ground speed) is much less than the air speed when a strong head wind is present. And for a VTOL aircraft hovering over a spot (zero ground speed) in a head wind, the two inertial terms are zero.

Therefore, the purpose of this Technical Comment is to point out a source of potential error in the analysis of VTOL aircraft dynamics—namely, not distinguishing between the U_0 values used on the left and right sides of Eqs. (1–6).

To illustrate the magnitude and assess the importance of such an error, consider the lateral-directional dynamics of the Doak VZ-4 tilt-duct aircraft hovering over a spot in a 59 fps head wind, which is the case considered in Ref. 1. After describing the aerodynamic forces and moments in terms of stability derivatives and the dependent variables, making the usual approximations that $r \cong \dot{\psi}$ and $p \cong \dot{\phi}$, and Laplace transforming, the factored characteristic equation is determined to be given correctly by Eq. (7).

$$(s - 0.08)(s + 1.59)(s + 0.0725 + j0.715)(s + 0.0725 - j0.715) = 0 \qquad (7)$$

By improperly retaining the mU_0r inertial term, the characteristic equation as obtained in Ref. 1 is

$$(s + 0.014)(s + 1.224)(s + 0.142 + j0.855)(s + 0.142 - j0.855) = 0 \qquad (8)$$

The complex roots in Eq. (7) yield a dutch-roll damping ratio of 0.101 and undamped natural frequency of 0.717 rad/sec, whereas those in Eq. (8) yield 0.164 and 0.868 rad/sec, respectively. The real roots in the two equations do not differ greatly; although Eq. (7) shows a slightly unstable system and Eq. (8) does not.

As an indication of how the error can influence transfer function numerator roots, the correct roll angle-to-aileron input numerator factored polynomial for this flight condition is given by expression (a), whereas Ref. 1 obtained expression (b).

$$0.504(s + 0.48)(s + 4.27) \qquad (a)$$

$$0.568(s + 0.407)(s + 1.712) \qquad (b)$$

Although the errors pointed out in these numerical results are not gross, neither are they negligible. It is hoped that this brief discussion will prevent future, potentially serious, errors of this type in dynamic analyses of VTOL aircraft.

Reference

[1] Smith, R. H., "VTOL Control Power Requirements Reapparaised," *Journal of Aircraft*, Vol. 3, No. 1, Jan.-Feb. 1966, pp. 11–17.

Received September 3, 1968.

* Associate Professor of Aeronautics, Astronautics and Engineering Sciences. Associate Fellow AIAA.

Reply by Author to R. L. Swaim

RALPH H. SMITH*
Princeton University, Princeton, N. J.

PROFESSOR Swaim is in error. The example he criticizes is, to the best of my knowledge, correct. In that example the aircraft motions were referred to a fixed air mass by defining U_0 as the trim airspeed. The zero ground speed was irrelevant to the subsequent dynamic analysis.

When done properly, the aircraft motions can be referred to any convenient Newtonian frame. In particular the motion can be referred to a ground-fixed reference system. For the cited example this would require redefining U_0 as the ground speed and setting it to zero, as Professor Swaim does. He fails to consider, however, that the aerodynamic force expressions (but not the forces) must also be altered. For example, $Y_v v$ in the first system becomes $Y_v(v - V_A\psi)$ in the second, where V_A is the mean wind speed with respect to the ground. The results in either system must, of course, be equivalent.

Professor Swaim may have been misled by the contrived nature of the example. My omission of a discussion of axis system details possibly compounded the confusion. Although these seemed superfluous at the time, in retrospect they would not have been. I apologize for this.

Received December 5, 1968; revision received December 13, 1968.

* Graduate Student, Department of Aerospace-Mechanical Sciences.